

Photon Structure Functions: Target Photon Mass Effects and QCD Corrections

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Abstract

We present a systematic analysis of the polarised and unpolarised processes $e^+ e^- \rightarrow e^+ e^- X$ in the deep inelastic limit and study the effects of target photon mass (virtuality) on the photon structure functions. The effect of target photon virtuality manifests as new singly polarised structure functions and also alters the physical interpretation of the unpolarised structure functions. The physical interpretation of these structure functions in terms of hadronic components is studied using the free field analysis. We also retrieve the real photon results in the limit the virtuality goes to zero. Assuming factorisation of the photon structure tensor, the relevant QCD corrections to the various twist two structure functions are evaluated.

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1 Introduction

The collision of photons at high energy electron-positron colliders is yet another comprehensive laboratory for testing Quantum Chromodynamics (QCD). The process $e^+ e^- \rightarrow e^+ e^- X$ (hadrons) (Fig. 1), at very high energies can be studied in terms of the hadronic structure of the photon. This process is dominated by the photon-photon \rightarrow hadrons subprocess. Ahmed and Ross [1] have studied the photon structure function by considering the $\gamma \gamma$ point scattering and using the Operator Production Expansion (OPE) to take the nonperturbative effects into account. The behaviour of this process in the context of perturbative QCD was first studied using the OPE by Witten [2], who showed that the unpolarised structure function F_2^γ increases as $\ln Q^2$, where $Q^2 \equiv -q^2$ (q is the momentum of the probing photon). The complete next to leading order correction to unpolarised structure functions and their phenomenological implications can be found in [3]. In the recent past the polarised structure function has attracted a lot of attention [4, 5, 6, 7]. In [4] the OPE analysis has been extended to the polarised sector, while in [5, 6] the first moment of polarised structure function has been evaluated and found to be zero for massless photons. The sensitivity of this sum rule due to the off-shell nature of the target photon has been addressed in [7].

Although the virtuality of the photon structure function had been studied [8] in the past, there has not been much emphasis on the effect arising out of the scalar polarisation ($\lambda = 0$) of the virtual photon ($k^2 < 0$, where k is momentum of the target photon). *The off-shell nature of the photon gives rise to scalar polarisation which in turn induces new structure functions to fully characterise the photon.* Some of these new structure functions are of leading twist (twist two) and would contribute in the Deep Inelastic Scattering (DIS) limit. Hence a comprehensive study of the photon structure functions at this stage including the target mass effects and QCD corrections is much awaited.

In the process we have considered, the subprocess photon-photon \rightarrow hadrons involves a large off-shell photon probing an off-shell target photon in the DIS limit. We *treat* the target photon as a composite object consisting of both hadronic and photonic components. In our analysis, we have treated the photon almost on par with the nucleon. This would imply that the probe photon would in fact see quarks, gluons (hadronic components) as well as photons (which can be produced at higher orders) in the off-shell target photon. All the higher order effects that go into the production of quarks, gluons and photons in the target photon are collectively treated as a blob as shown in the Fig. 4. Hence we are now in a position to utilise all the machinery that

is used in the lepton-nucleon DIS. Due to the off-shell nature of the target photon, additional polarisation of the target gives rise to new structure functions to completely characterise the process. These new structure functions are the singly polarised structure functions. In the context of the photon, these new structure functions arising due to scalar polarisation of the target photon (mass effect) are being considered for the first time. To understand the structure functions in terms of the hadronic components we perform a free field analysis of this process. In doing so we arrive at various sum rules and relations among the various photon structure functions. These structure functions are related to the photon matrix elements of some bilocal operators similar to those one comes across in DIS and Drell-Yan. We assume a factorisation of hard and soft parts in order to calculate the hadronic and photonic contributions to the cross section. Our approach is different from the previous analysis which usually uses either the parton model picture or OPE.

The rest of the paper is organised as follows. In section 2 we study the effects of the virtuality of the target photon on the photon structure tensor and hence the polarised and unpolarised cross sections. In section 3 we perform a free field analysis to study the physical interpretation of the new structure functions and demonstrate how the unpolarised structure functions are altered due to the virtuality of the photon. In section 4 we use the factorisation method to evaluate the QCD corrections to the new photon structure functions. Finally we conclude in section 5.

2 Target photon mass effects

Consider the process $e^-(p_1, s_1) e^+(p_2, s_2) \rightarrow e^-(p'_1) e^+(p'_2) X(p_n)$, where s_1, s_2 are the polarisation vectors of the leptons and X represents the final state hadrons. In the cm frame, the momenta of the in coming and out going particles are parametrised as $p_1 = (E_1, 0, 0, E_1)$, $p_2 = (E_1, 0, 0, -E_1)$ and $p'_1 = E'_1(1, 0, \sin \theta_1, \cos \theta_1)$, $p'_2 = E'_2(1, 0, -\sin \theta_2, -\cos \theta_2)$ respectively. $\theta_{1,2}$ are the scattering angles of out going leptons with respect to the beam axis. This process (Fig. 1) at very high energies is dominated by the photon-photon \rightarrow hadrons subprocess. In our analysis we consider a probe photon $\gamma^*(q)$ probing a target photon $\Gamma(k)$ in the DIS limit. We keep the target photon in general to be nonperturbative and study the effect of virtuality on the photon structure functions. The total cross section for this process is given by

$$d\sigma = \frac{1}{8E_1^2} \frac{d^3p'_1}{(2\pi)^3 2E'_1} \frac{d^3p'_2}{(2\pi)^3 2E'_2} \prod_{i=1}^n \frac{d^3p_i}{(2\pi)^3 2E_i} |T_n|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p'_1 - p'_2 - p_n) , \quad (1)$$

where T_n is the transition amplitude and n refers to the final state. The double differential cross section is found to be

$$\begin{aligned} \frac{d\sigma^{s_1 s_2}}{dx \, dQ^2} &= \frac{\alpha^3}{4x s^2 Q^2} \int \frac{d\kappa^2}{\kappa^2} \int_x^1 \frac{dy}{y^2} L^{\mu\mu'}(q, p_1, s_1) L^{\nu\nu'}(k, p_2, s_2) \\ &\times \sum_{\lambda=0, \pm 1} g^{\lambda\lambda} \epsilon_\nu^*(k, \lambda) \epsilon_{\nu'}(k, \lambda) W_{\mu\mu'}^\Gamma(q, k, \lambda) , \end{aligned} \quad (2)$$

where $\alpha = e^2/4\pi$, s is the centre of mass energy of incoming leptons, $q = p_1 - p'_1$ and $k = p_2 - p'_2$ are the momenta of the probe and target photons with invariant mass $Q^2 = -q^2$ and $\kappa^2 = -k^2$ respectively. The Björken variable with respect to the target positron is defined as $x \equiv Q^2/2\tilde{\nu}$, where $\tilde{\nu} = p_2 \cdot q$ and that with respect to the target photon is $y \equiv Q^2/2\nu$, where $\nu = k \cdot q$. The DIS limit corresponds to $Q^2, \tilde{\nu}, \nu \rightarrow \infty$ with x and y fixed. Further we are interested in the region $\kappa^2 \ll Q^2$ and hence do not consider terms of the order $\mathcal{O}(\kappa^2/Q^2)$. However we would like to address the effects of scalar polarisation on the photon structure functions. The vector $\epsilon^\mu(k, \lambda)$ is the polarisation vector of the target photon with polarisation λ . Photons with $k^2 < 0$, are characterised by $\lambda = 0$ (scalar) in addition to $\lambda = \pm 1$ (transverse) polarisation states. The polarisation vectors corresponding to these states satisfy the relation: $\sum_{\lambda=0, \pm 1} g^{\lambda\lambda} \epsilon_\mu^*(k, \lambda) \epsilon_\nu(k, \lambda) = k^2 g_{\mu\nu} - k_\mu k_\nu$ with $\epsilon(k, \lambda) \cdot k = 0$. The lepton tensor $L^{\mu\nu}(q, p, s_i)$ is generically defined as

$$L^{\mu\nu}(q, p, s_i) = 4 p^\mu p^\nu - 2 (p^\mu q^\nu + p^\nu q^\mu) + 2 p \cdot q g^{\mu\nu} - 2i \epsilon^{\mu\nu\alpha\beta} s_\alpha^i q_\beta , \quad (3)$$

where $s_i \cdot p = 0$ and $s_i^2 = -m_e^2$. The cross section (eqn. (2)) can be written in terms of the electron structure tensor $W_{\mu\mu'}^e(q, p_2, s_2)$ as

$$\frac{d\sigma^{s_1 s_2}}{dx \, dQ^2} = \frac{\alpha^2}{4x^2 s^2 Q^2} L^{\mu\mu'}(q, p_1, s_1) W_{\mu\mu'}^e(q, p_2, s_2) , \quad (4)$$

where $W_{\mu\mu'}^e(x, Q^2, s_2)$ is

$$W_{\mu\mu'}^e(x, Q^2, s_2) = \alpha x \int \frac{d\kappa^2}{\kappa^2} \int_x^1 \frac{dy}{y^2} L^{\nu\nu'}(k, p_2, s_2) \sum_{\lambda=0, \pm 1} g^{\lambda\lambda} \epsilon_\nu^*(k, \lambda) \epsilon_{\nu'}(k, \lambda) W_{\mu\mu'}^\Gamma(q, k, \lambda) . \quad (5)$$

Due to the presence of the nonperturbative photon tensor $W_{\mu\nu}^\Gamma(q, k, \lambda)$, the electron structure tensor $W_{\mu\nu}^e(x, Q^2, s_2)$ can at best be parametrised as a spin 1/2 target in terms of vectors q, p_2, s_2 , subject to the symmetries as

$$\begin{aligned} W_{\mu\nu}^e(x, Q^2, s_2) &= F_1^e(x, Q^2) G_{\mu\nu} + F_2^e(x, Q^2) \frac{\tilde{R}_\mu \tilde{R}_\nu}{\tilde{\nu}} \\ &+ \frac{i}{\tilde{\nu}^2} \epsilon_{\mu\nu\alpha\beta} \left(g_1^e(x, Q^2) \tilde{\nu} q^\alpha s_2^\beta + g_2^e(x, Q^2) q^\alpha \left(\tilde{\nu} s_2^\beta - s_2 \cdot q p_2^\beta \right) \right) . \end{aligned} \quad (6)$$

Here $F_{1,2}^e(x, Q^2)$ and $g_{1,2}^e(x, Q^2)$ are the unpolarised and polarised electron structure functions respectively. The tensor coefficients in the above equation are defined as

$$G_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} , \quad \tilde{R}^\mu = p_2^\mu - \frac{\tilde{\nu}}{q^2} q^\mu . \quad (7)$$

The photon structure tensor $W_{\mu\nu}^\Gamma(q, k, \lambda)$ is the imaginary part of the forward amplitude $\gamma^*(q, \lambda') \Gamma(k, \lambda) \rightarrow \gamma^*(q, \lambda') \Gamma(k, \lambda)$ and can be defined as the Fourier transform of the commutator of electromagnetic (*em*) currents $J_\mu(\xi)$ sandwiched between target photon states, as

$$W_{\mu\nu}^\Gamma(k, q, \lambda) = \frac{1}{2\pi} \int d^4\xi e^{-iq \cdot \xi} \langle \Gamma(k, \epsilon^*(\lambda)) | [J_\mu(\xi), J_\nu(0)] | \Gamma(k, \epsilon(\lambda)) \rangle_c , \quad (8)$$

where the subscript *c* denotes the connected part. For a real photon $\Gamma(\lambda) \rightarrow \gamma(\lambda)$, the polarisation states are $\lambda = \pm 1$ while the probe photon polarisation $\lambda' = 0, \pm 1$, since $q^2 \neq 0$ and is very large in the DIS limit. The number of independent helicity amplitudes is four, corresponding to the four structure functions $F_{1,2}^\gamma(y, Q^2)$ and $g_{1,2}^\gamma(y, Q^2)$. The virtual photon ($\kappa^2 \neq 0$) on the other hand is characterised by polarisation states $\lambda = 0, \pm 1$. Enumerating the number of independent helicity amplitudes subject to parity and time reversal invariance gives eight independent amplitudes. Hence the photon structure tensor $W_{\mu\nu}^\Gamma(k, q, \lambda)$ can be parametrised in a gauge invariant way in terms of the eight structure functions using general symmetry arguments such as time reversal invariance, parity, hermiticity and current conservation. Thus

$$\begin{aligned} W_{\mu\nu}^\Gamma(y, Q^2, \kappa^2, \lambda) = & \frac{1}{\kappa^4} \left\{ F_1^\Gamma(y, Q^2, \kappa^2) G_{\mu\nu} + F_2^\Gamma(y, Q^2, \kappa^2) \frac{R_\mu R_\nu}{\nu} \right. \\ & + b_1^\Gamma(y, Q^2, \kappa^2) r_{\mu\nu} + b_2^\Gamma(y, Q^2, \kappa^2) s_{\mu\nu} \\ & + b_3^\Gamma(y, Q^2, \kappa^2) t_{\mu\nu} + b_4^\Gamma(y, Q^2, \kappa^2) u_{\mu\nu} \\ & \left. + \frac{i}{\nu^2} \epsilon_{\mu\nu\lambda\rho} \left(g_1^\Gamma(y, Q^2, \kappa^2) \nu q^\lambda s^\rho + g_2^\Gamma(y, Q^2, \kappa^2) q^\lambda (\nu s^\rho - s \cdot q k^\rho) \right) \right\} , \end{aligned} \quad (9)$$

where s_μ is the spin vector of the target photon, $R_\mu = x \tilde{R}_\mu / y$ and the various tensors are defined as

$$\begin{aligned} r_{\mu\nu} &= \frac{1}{4} \left(\frac{k \cdot E^* k \cdot E}{\kappa^4} - \bar{\alpha}^2 \right) G_{\mu\nu} , \quad s_{\mu\nu} = \frac{1}{4\nu} \left(\frac{k \cdot E^* k \cdot E}{\kappa^4} - \bar{\alpha}^2 \right) R_\mu R_\nu , \\ t_{\mu\nu} &= -\frac{1}{8\nu} \left(\frac{k \cdot E^*}{\kappa^2} (R_\mu E_\nu + R_\nu E_\mu) + \frac{k \cdot E}{\kappa^2} (R_\mu E_\nu^* + R_\nu E_\mu^*) - 4(1 - \bar{\alpha}^2) R_\mu R_\nu \right) , \\ u_{\mu\nu} &= \frac{1}{4\nu} \left(E_\mu^* E_\nu + E_\nu^* E_\mu + 2\kappa^2 G_{\mu\nu} + 2(1 - \bar{\alpha}^2) R_\mu R_\nu \right) , \\ E_\mu &= \epsilon_\mu - \frac{q \cdot \epsilon}{\nu} k_\mu , \quad \bar{\alpha}^2 = 1 - \frac{\kappa^2 Q^2}{\nu^2} , \quad s^\mu \equiv \frac{i}{\kappa^2} \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* \epsilon_\alpha k_\beta . \end{aligned} \quad (10)$$

This is the standard decomposition of a massive spin one composite target [9]. The additional input that has gone in the case of the virtual photon is the manifest gauge invariance of the tensor coefficients. The four new structure functions $b_{1-4}^\Gamma(y, Q^2, \kappa^2)$ are due to the off-shell nature ($\lambda = 0$, scalar polarisation ($k^2 < 0$)) of the target photon. The tensor coefficients of these additional structure functions vanish when the target photon polarisation is summed, and survive when the target is polarised while the probe polarisation is summed. Hence these structure functions are called the singly polarised structure functions. This singly polarised nature is characteristic of a spin one target [9]. In the context of photon structure functions which are realised in a $e^+ e^- \rightarrow e^+ e^- X$ process, the singly polarised part does not manifest itself as in a spin one target, but turns out to be a part of the unpolarised cross section.

The unpolarised, singly polarised and the polarised sectors which characterise the virtual photon tensor $W_{\mu\nu}^\Gamma(y, Q^2, \kappa^2)$ in eqn. (9) are all independent and can be extracted using the following combinations of polarisation states

$$\begin{aligned}\overline{W}_{\mu\nu} &= \sum_{\lambda=0,\pm 1} g^{\lambda\lambda} W_{\mu\nu}^\Gamma(\lambda), \\ \delta W_{\mu\nu} &= \sum_{\lambda=0,\pm 1} C(\lambda) W_{\mu\nu}^\Gamma(\lambda), \\ \Delta W_{\mu\nu} &= \sum_{\lambda=0,\pm 1} C'(\lambda) W_{\mu\nu}^\Gamma(\lambda),\end{aligned}\tag{11}$$

where $g^{\lambda\lambda}$ is the metric tensor, $C(\lambda) = 2$ for $\lambda = 0$; -1 for $\lambda = \pm 1$ and $C'(\lambda) = 0$ for $\lambda = 0$; ± 1 for $\lambda = \pm 1$. The polarised combination $\Delta W_{\mu\nu}$ is same as the real photon combination. This is due to the fact that the virtuality of the photon induces the $\epsilon_\mu(\lambda = 0)$ polarisation state and this does not contribute to the antisymmetric polarised part $\Delta W_{\mu\nu}$. In contrast the unpolarised combination $\overline{W}_{\mu\nu}$ is altered due to the virtuality as it is a sum over all the polarisation states of the virtual photon. For the real photon the $\epsilon_\mu(\lambda = 0)$ polarisation state would be absent and hence the unpolarised $\overline{W}_{\mu\nu}$ and singly polarised $\delta W_{\mu\nu}$ combination reduces to the real photon unpolarised combination which is the sum of $\lambda = \pm 1$ polarisation states.

The unpolarised and polarised cross sections for $e^+e^- \rightarrow e^+e^-X$ can be derived using eqns. (4,5) and eqn. (9). The details of this derivation are given in the Appendix. The unpolarised cross section is given by

$$\begin{aligned}\frac{d\sigma^{\uparrow\uparrow+\uparrow\downarrow}}{dx dQ^2} &= \frac{\alpha^3}{xs^2Q^2} L_{sym}^{\mu\mu'}(q, p_1, \uparrow) \int \frac{d\kappa^2}{\kappa^2} \int_x^1 \frac{dy}{y^2} \kappa^4 \left\{ -\frac{y}{x} \overline{P}_{\gamma e} \left(\frac{x}{y} \right) \overline{W}_{\mu\mu'}^\Gamma(y, Q^2, \kappa^2) \right. \\ &\quad \left. + \frac{y}{2x} \delta P_{\gamma e} \left(\frac{x}{y} \right) \delta W_{\mu\mu'}^\Gamma(y, Q^2, \kappa^2) \right\},\end{aligned}\tag{12}$$

where \uparrow (\downarrow) denotes the polarisation of an electron along (opposite) to the beam direction and $L_{sym}^{\mu\mu'}(q, p_1, \uparrow)$ is the symmetric part of the lepton tensor eqn. (3). In the above equation, the first term in the curly bracket corresponds to unpolarised structure function and the second term to the singly polarised structure functions. The modified splitting functions ($k^2 \neq 0$) are given by

$$\bar{P}_{\gamma e} \left(\frac{x}{y} \right) = \frac{y}{x} \left(2 - 2\frac{x}{y} + \frac{x^2}{y^2} \right) - 2 \frac{y}{x} \left(1 - \frac{x}{y} \right) , \quad (13)$$

$$\delta P_{\gamma e} \left(\frac{x}{y} \right) = \frac{y}{x} \left(2 - 2\frac{x}{y} + \frac{x^2}{y^2} \right) - 4 \frac{y}{x} \left(1 - \frac{x}{y} \right) . \quad (14)$$

The first term in the above equations is the usual Weizsäcker-Williams splitting function arising from the splitting of e^+ into transverse photons ($k^2 = 0$). The additional term arises from the emission of scalar polarised photon. The unpolarised splitting function $\bar{P}_{\gamma e} \propto \sum_{\lambda=0\pm 1} g^{\lambda\lambda} \epsilon_\mu^*(\lambda) \epsilon_\nu(\lambda) L^{\mu\nu}(k, p_2, s_2)$ and the singly polarised $\delta P_{\gamma e} \propto \sum_{\lambda=0\pm 1} C(\lambda) \epsilon_\mu^*(\lambda) \epsilon_\nu(\lambda) L^{\mu\nu}(k, p_2, s_2)$. Hence there is a neat factorisation of the cross section (eqn. (12)) in terms of the combination of polarisation states used to extract unpolarised and singly polarised structure functions. The polarised cross section is given by

$$\frac{d\sigma^{\uparrow\uparrow-\uparrow\downarrow}}{dx dQ^2} = -\frac{\alpha^3}{x s^2 Q^2} L_{asym}^{\mu\mu'}(q, p_1, \uparrow) \int \frac{d\kappa^2}{\kappa^2} \int_x^1 \frac{dy}{y^2} \kappa^4 \frac{y}{x} \Delta P_{\gamma e} \left(\frac{x}{y} \right) \Delta W_{\mu\mu'}^\Gamma(y, Q^2, \kappa^2) , \quad (15)$$

where $L_{asym}^{\mu\mu'}(q, p_1, \uparrow)$ is the antisymmetric part of eqn. (3) and the polarised splitting function is

$$\Delta P_{\gamma e} \left(\frac{x}{y} \right) = 2 - \frac{x}{y} .$$

Note that the splitting function $\Delta P_{\gamma e}$ is the same as in the real case. As expected the additional scalar polarisation of the photon does not contribute to the polarised cross section.

The electron structure functions can now be related to the photon structure functions by substituting the photon structure tensor eqn. (9) in eqns. (12,15) and comparing it with the total cross section eqn. (4) after substituting eqn. (6). Hence we get

$$\begin{aligned} F_1^e(x, Q^2) &= 2\alpha \int \frac{d\kappa^2}{\kappa^2} \int_x^1 \frac{dy}{y} \left[\bar{P}_{\gamma e} \left(\frac{x}{y} \right) F_1^\Gamma(y, Q^2, \kappa^2) \right. \\ &\quad \left. - \frac{1}{2} \delta P_{\gamma e} \left(\frac{x}{y} \right) \left(\frac{\bar{\alpha}^2}{2} b_1^\Gamma(y, Q^2, \kappa^2) - \frac{\kappa^2}{2\nu} b_4^\Gamma(y, Q^2, \kappa^2) \right) \right] , \\ F_2^e(x, Q^2) &= 2\alpha \int \frac{d\kappa^2}{\kappa^2} \int_x^1 \frac{dy}{y} \frac{x}{y} \left[\bar{P}_{\gamma e} \left(\frac{x}{y} \right) F_2^\Gamma(y, Q^2, \kappa^2) \right. \\ &\quad \left. - \frac{1}{2} \delta P_{\gamma e} \left(\frac{x}{y} \right) \left(\frac{\bar{\alpha}^2}{2} b_2^\Gamma(y, Q^2, \kappa^2) - (1 - \bar{\alpha}^2) b_3^\Gamma(y, Q^2, \kappa^2) \right) \right] , \end{aligned} \quad (16)$$

$$+ \frac{1 - \bar{\alpha}^2}{2\bar{\alpha}^2} (1 - 2\bar{\alpha}^2) b_4^\Gamma(y, Q^2, \kappa^2) \Big] , \quad (17)$$

$$g_1^e(x, Q^2) = 4\alpha \int \frac{d\kappa^2}{\kappa^2} \int_x^1 \frac{dy}{y} \left[\Delta P_{\gamma e} \left(\frac{x}{y} \right) g_1^\Gamma(y, Q^2, \kappa^2) \right] . \quad (18)$$

Note that the unpolarised electron structure functions are related to the singly polarised structure functions of photon. This extra contribution comes from the scalar polarisation (virtuality) of the photon which is also reflected in the modified splitting functions. To leading order in the DIS limit, the unpolarised electron structure functions $F_{1,2}^e(x, Q^2)$ are modified only by the twist two singly polarised structure functions $b_{1,2}^\Gamma(y, Q^2, \kappa^2)$ respectively. The other singly polarised structure functions $b_{3,4}^\Gamma(y, Q^2, \kappa^2)$ do not contribute in the DIS limit as their coefficients are of the form $(1 - \bar{\alpha}^2)$ which in this limit goes like κ^2/Q^2 . This naively counts the twist of the new structure functions i.e., $b_{1,2}^\Gamma(y, Q^2, \kappa^2)$ are twist two and $b_{3,4}^\Gamma(y, Q^2, \kappa^2)$ are twist four structure functions. Higher twist contributions go as powers of κ^2/Q^2 and hence the structure functions $b_{3,4}^\Gamma(y, Q^2, \kappa^2)$ are not pursued any further. It is imperative at this stage to show that in the limit $\kappa^2 \rightarrow 0$, we can retrieve the real photon results. At this stage we are not in a position to restore the real photon results as we do not know the behaviour of the structure function in the limit $\kappa^2 \rightarrow 0$. So in the next section we show that our present analysis is consistent with the earlier works on photon structure function in the $\kappa^2 \rightarrow 0$ limit (real photon). We also show that two of the new structure functions ($b_{1,2}^\Gamma(y, Q^2, \kappa^2)$) are of twist two. The polarised structure function $g_1^e(x, Q^2)$ on the other hand is unaffected by the virtuality of the photon.

In terms of the above relations we can compute the unpolarised and polarised cross sections and these are given by

$$\frac{d\sigma^{\uparrow\uparrow\uparrow\downarrow}}{dx \, dQ^2} = \frac{\alpha^2}{x^2 \, s \, Q^2} \left\{ F_1^e(x, Q^2) \frac{Q^2}{s} - F_2^e(x, Q^2) \left(1 - \frac{xs}{Q^2} \right) \right\} , \quad (19)$$

$$\frac{d\sigma^{\uparrow\uparrow-\uparrow\downarrow}}{dx \, dQ^2} = 2 \frac{\alpha^2}{x \, s \, Q^2} g_1^e(x, Q^2) \left(1 - \frac{Q^2}{2xs} \right) . \quad (20)$$

Now by substituting for the electron structure functions from eqns. (16–18) the n^{th} moment of the above differential cross section can be related to the $(n-1)^{\text{th}}$ moment of the photon structure functions.

In this section we have shown that in the DIS limit, the virtuality of the target photon manifests as new singly polarised twist two structure functions $b_{1,2}^\Gamma(x, Q^2, \kappa^2)$ and in addition the $e \rightarrow \gamma$ splitting function also gets modified. Our next task is to understand these new structure functions in terms of the parton distributions.

3 Free field analysis

To understand the hadronic structure of the photon structure functions, we make use of the free field analysis *akin* to the one used in the case of nucleon structure function. We present a systematic study of these new structure functions i.e., their twist structure and their physical interpretation in terms of parton content. The twist analysis is necessary as the partonic interpretation is possible only for twist two operators. This is done using free field analysis. We will show from the twist analysis that $b_{1,2}^\Gamma(y, Q^2, \kappa^2)$ have definite parton model interpretation as they are related to twist two operators. We will also show as a by product that in the $\kappa^2 \rightarrow 0$ limit, we can reproduce the real photon results.

In this analysis one assumes the *em* current J_μ to be made of free quark currents while the photon states are nonperturbative. This analysis should therefore not be confused with the conventional Quark Parton Model in the context of photon structure function wherein the photon is directly coupled to the charge of the bare quark and is realised only at large κ^2 . In the DIS limit, the leading contribution to the commutator in eqn. (8) comes from the light cone region $\xi^2 \rightarrow 0$. Noting that the commutator is proportional to the imaginary part of the time ordered product of currents and using Wick's expansion, we get

$$[J_\mu(\xi), J_\nu(0)] = \frac{\delta^{(1)}(\xi^2) \xi^\lambda}{\pi} \left\{ \sigma_{\mu\lambda\nu\rho} O_{(-)}^\rho(\xi) - i\epsilon_{\mu\lambda\nu\rho} O_{(+)5}^\rho(\xi) \right\}, \quad (21)$$

where

$$\begin{aligned} \delta^{(1)}(\xi^2) &= \frac{\partial}{\partial \xi^2} \delta(\xi^2), \\ \sigma_{\mu\lambda\nu\rho} &= g_{\mu\lambda} g_{\nu\rho} - g_{\mu\nu} g_{\lambda\rho} + g_{\mu\rho} g_{\lambda\nu}, \\ O_{(\pm)}^\rho(\xi) &= : \bar{\psi}(\xi) \gamma^\rho \psi(0) \pm \bar{\psi}(0) \gamma^\rho \psi(\xi) : , \\ O_{(\pm)5}^\rho(\xi) &= : \bar{\psi}(\xi) \gamma^\rho \gamma_5 \psi(0) \pm \bar{\psi}(0) \gamma^\rho \gamma_5 \psi(\xi) : , \end{aligned} \quad (22)$$

where the symbol $: :$ implies normal ordering of operators. Substituting this in eqn. (8), performing the $d^4\xi$ integral and comparing the tensor coefficients with eqn. (9), we relate the structure functions to various scaling functions as given in Table 1. $\tilde{A}(y)$, $\tilde{B}(y)$ and $\tilde{C}(y)$ in the Table 1 are defined as

$$\int dz' e^{iz'k \cdot \xi} \begin{pmatrix} \tilde{A}(z') \\ \tilde{B}(z') \\ \tilde{C}(z') \end{pmatrix} = \begin{pmatrix} A(k \cdot \xi) \\ B(k \cdot \xi) \\ C(k \cdot \xi) \end{pmatrix} = \sum_n \frac{1}{(n+1)!} k \cdot \xi^{n-1} \begin{pmatrix} (n+1)A_n k \cdot \xi \\ B_n \\ C_n k \cdot \xi \end{pmatrix}, \quad (23)$$

where A_n , B_n , C_n are the expansion coefficients of local photon matrix elements given below

$$\langle \Gamma(k, \epsilon^*) | O_{(-)}^{\rho\mu_1 \dots \mu_n}(0) | \Gamma(k, \epsilon) \rangle = 2A_n \mathcal{S}(k^\rho k^{\mu_1} \dots k^{\mu_n}) + B_n \mathcal{S}[(\epsilon^{\rho*} \epsilon^{\mu_1} + k^\rho k^{\mu_1}) k^{\mu_2} \dots k^{\mu_n}] , \quad (24)$$

$$\langle \Gamma(k, \epsilon^*) | O_{(+5)}^{\rho\mu_1 \dots \mu_n}(0) | \Gamma(k, \epsilon) \rangle = C_n \mathcal{S}(s^\rho k^{\mu_1} \dots k^{\mu_n}) . \quad (25)$$

Here A_n, B_n, C_n are functions of Lorentz invariants such as k^2, s^2 etc. \mathcal{S} denotes symmetrisation with respect to all indices. This is done to ensure that only leading twist operators contribute. The matrix element A_n contributes to the unpolarised part, B_n to the singly polarised part and C_n to the polarised part. From Table 1 it is clear that both the unpolarised structure functions ($F_{1,2}^\Gamma(y)$) and the singly polarised structure functions ($b_{1,2}^\Gamma(y)$) satisfy Callan-Gross relation. In addition we find that $b_3^\Gamma(y)$ and $b_4^\Gamma(y)$ are related to $b_2^\Gamma(y)$ by the following relations:

$$b_4^\Gamma(y) = - \int_y^1 \frac{dy'}{y'} b_3^\Gamma(y') , \quad (26)$$

$$b_3^\Gamma(y) = - \int_y^1 \frac{dy'}{y'} b_2^\Gamma(y') . \quad (27)$$

The physical interpretation of these structure functions can be given using the above free field analysis. This is done by substituting the current commutator eqn. (21) in eqn. (8) and performing only the $d\xi^+$ and $d\xi_\perp$ integrals, where $\xi^\pm = (\xi^0 \pm \xi^3)/\sqrt{2}$, $\xi_\perp = (\xi^1, \xi^2)$. The unpolarised, singly polarised and polarised structure functions can be separated using the combinations of the polarisation states defined in eqn. (11). Using appropriate projection operators for the various structure functions, we get

$$F_1^\Gamma(y) = \frac{1}{4\pi} \int d\xi^- e^{-iyk^+ \xi^-} \langle \Gamma(k) | \overline{O}_{(-)}^+(0, \xi^-, 0_\perp) | \Gamma(k) \rangle , \quad (28)$$

$$b_1^\Gamma(y) = \frac{1}{2\pi} \int d\xi^- e^{-iyk^+ \xi^-} \langle \Gamma(k, \epsilon^*) | \delta O_{(-)}^+(0, \xi^-, 0_\perp) | \Gamma(k, \epsilon) \rangle , \quad (29)$$

$$g_1^\Gamma(y) = \frac{1}{4\pi} \int d\xi^- e^{-iyk^+ \xi^-} \langle \Gamma(k, \epsilon^*) | \Delta O_{(+5)}^+(0, \xi^-, 0_\perp) | \Gamma(k, \epsilon) \rangle , \quad (30)$$

where the superscript $+$ denotes the light cone variable. The matrix elements in the above equations are defined as

$$\begin{aligned} \langle \Gamma(k) | \overline{O}_{(-)}^+(0, \xi^-, 0_\perp) | \Gamma(k) \rangle &= \sum_{\lambda=0, \pm 1} g^{\lambda\lambda} \langle \Gamma(k, \epsilon^*(\lambda)) | O_{(-)}^+(0, \xi^-, 0_\perp) | \Gamma(k, \epsilon(\lambda)) \rangle , \\ \langle \Gamma(k, \epsilon^*) | \delta O_{(-)}^+(0, \xi^-, 0_\perp) | \Gamma(k, \epsilon) \rangle &= \sum_{\lambda=0, \pm 1} C(\lambda) \langle \Gamma(k, \epsilon^*(\lambda)) | O_{(-)}^+(0, \xi^-, 0_\perp) | \Gamma(k, \epsilon(\lambda)) \rangle , \\ \langle \Gamma(k, \epsilon^*) | \Delta O_{(+5)}^+(0, \xi^-, 0_\perp) | \Gamma(k, \epsilon) \rangle &= \sum_{\lambda=0, \pm 1} C'(\lambda) \langle \Gamma(k, \epsilon^*(\lambda)) | O_{(+5)}^+(0, \xi^-, 0_\perp) | \Gamma(k, \epsilon(\lambda)) \rangle . \end{aligned} \quad (31)$$

Using these matrix elements, the structure functions can be interpreted in terms of the probability of finding a quark of helicity h in a target photon of helicity λ denoted by $f_{q(h)/\Gamma(\lambda)}$ where

$$f_{\frac{q(h)}{\Gamma(\epsilon(\lambda))}}(z, \mu^2, \kappa^2) = \frac{1}{4\pi} \int d\xi^- e^{-iz} \xi^- k^+ \langle \Gamma(k, \epsilon^*(\lambda)) | \bar{\psi}(0, \xi^-, 0_\perp) \gamma^+ \Lambda_h \psi(0) | \Gamma(k, \epsilon(\lambda)) \rangle, \quad (32)$$

where $\Lambda_h = (1 + h \gamma_5)/2$. In terms of $f_{a(h)/\Gamma(\lambda)}$ the structure functions are of the form

$$\begin{aligned} F_1^\Gamma(y) &= f_{\frac{a(\uparrow)}{\Gamma(0)}} - f_{\frac{a(\uparrow)}{\Gamma(1)}} - f_{\frac{a(\downarrow)}{\Gamma(1)}}, \\ b_1^\Gamma(y) &= 2 \left(2f_{\frac{a(\uparrow)}{\Gamma(0)}} - f_{\frac{a(\uparrow)}{\Gamma(1)}} - f_{\frac{a(\downarrow)}{\Gamma(1)}} \right), \\ g_1^\Gamma(y) &= f_{\frac{a(\uparrow)}{\Gamma(1)}} - f_{\frac{a(\downarrow)}{\Gamma(1)}}. \end{aligned} \quad (33)$$

Interestingly, both $F_{1,2}^\Gamma(y, Q^2, \kappa^2)$ and $b_{1,2}^\Gamma(y, Q^2, \kappa^2)$ carry nontrivial information about the photon i.e., parton content of the scalar polarised photon ($\epsilon_\mu(\lambda = 0)$). This is absent in the case of real photon as it has only transverse polarisation states ($\epsilon_\mu(\lambda = \pm 1)$). Knowing $b_1^\Gamma(y, Q^2, \kappa^2)$ and $F_1^\Gamma(y, Q^2, \kappa^2)$ we can project out the parton content of the scalar polarised photon. The authors of Ref. [9] expect that for ρ meson $b_1^\Gamma \sim \mathcal{O}(F_1^\Gamma)$ and hence in the region $0 < \kappa^2 < \Lambda^2$ (where Λ is the QCD scale parameter) the scalar contribution may be substantial.

Now that we know the hadronic structure of the photon in terms of the helicity states eqn. (33), we are in a position to take the $\kappa^2 \rightarrow 0$ limit and check if we could reproduce the real photon results. The matrix elements (eqn. (33)) are perturbatively calculable in the region $\Lambda^2 \ll \kappa^2 \ll Q^2$. We keep quark masses nonzero to exhibit κ^2 dependence of these operator matrix elements. To regulate ultraviolet divergence we use dimensional regularisation and \overline{MS} scheme is used to renormalise at the scale μ_R^2 . Hence the probability to find a quark with momentum fraction y inside transversely polarised ($\epsilon_\mu(\lambda = \pm 1)$) and scalar polarised ($\epsilon_\mu(\lambda = 0)$) photons is

$$\begin{aligned} f_{\frac{q(\uparrow)}{\gamma(\lambda=\pm 1)}}(y, \mu_R^2, \kappa^2) &= \frac{\alpha}{4\pi} \left[(2y^2 - 2y + 1) \ln \frac{\mathcal{M}_\gamma^2}{\mu_R^2} - \frac{2m^2 y(1-y)}{\mathcal{M}_\gamma^2} - \frac{\kappa^2 y(1-y)}{\mathcal{M}_\gamma^2} + 2y(1-y) \right], \\ f_{\frac{q(\uparrow)}{\gamma(\lambda=0)}}(y, \mu_R^2, \kappa^2) &= -\frac{\alpha}{2\pi} \frac{\kappa^2}{\mathcal{M}_\gamma^2} y^2 (1-y)^2, \end{aligned} \quad (34)$$

where $\mathcal{M}_\gamma^2 = m^2 + \kappa^2 y(1-y)$. It is clear from the above equation that the regulator (quark mass) going to zero, the quark content of the scalar polarised photon is $\alpha y(y-1)/(2\pi)$. The above exercise proves that the new structure functions $b_{1,2}^\Gamma(y, Q^2, \kappa^2)$ are of twist two, as the scalar polarised photon contribution goes as $\kappa^2/\mathcal{M}_\gamma^2$. Note that it is not a power of κ^2/Q^2 as in the case of higher twist contributions. In the limit $\kappa^2 \rightarrow 0$, $f_{q/\Gamma(\epsilon(0))}$ goes to zero and hence we

have $b_{1,2}^\gamma = 2F_{1,2}^\gamma$. By taking the limit $\kappa^2 \rightarrow 0$, we would be approaching the nonperturbative region and hence the limit is in fact nontrivial. But since a real photon ($k^2 = 0$) is characterised only by $\lambda = \pm 1$, we expect the scalar contributions should go to zero in the limit $\kappa^2 \rightarrow 0$. Now taking the limit $\kappa^2 \rightarrow 0$ in eqns. (16, 17), replacing $\Gamma \rightarrow \gamma$ and equating $b_{1,2}^\gamma = 2F_{1,2}^\gamma$, we have

$$F_1^e(x, Q^2) = \alpha \int \frac{d\kappa^2}{\kappa^2} \int_x^1 \frac{dy}{y} P_{\gamma e} \left(\frac{x}{y} \right) F_1^\gamma(y, Q^2), \quad (35)$$

$$F_2^e(x, Q^2) = \alpha \int \frac{d\kappa^2}{\kappa^2} \int_x^1 \frac{dy}{y} \frac{x}{y} P_{\gamma e} \left(\frac{x}{y} \right) F_2^\gamma(y, Q^2), \quad (36)$$

where $P_{\gamma e}$ is the usual Weizäcker-William splitting function for a real photon,

$$P_{\gamma e} \left(\frac{x}{y} \right) = \frac{y}{x} \left(2 - 2\frac{x}{y} + \frac{x^2}{y^2} \right). \quad (37)$$

Thus we have reproduced the real photon results in the limit $\kappa^2 \rightarrow 0$.

In this section we have used the free field analysis to study the physical interpretation of the new singly polarised structure function and showed how the virtuality of the target photon alters the partonic interpretation of the unpolarised structure function. Next we study the QCD corrections to these new structure functions and the modified unpolarised structure functions.

4 QCD corrections: Factorisation method

From the free field analysis of the previous section we find that the hadronic structure of the virtual photon is modified by scalar polarisation effects. This modifies the unpolarised structure functions $F_{1,2}^\Gamma(y, Q^2, \kappa^2)$ while the polarised structure function $g_1^\Gamma(y, Q^2, \kappa^2)$ is unaltered. Further the virtuality of the photon gives rise to additional singly polarised structure functions $b_{1-4}^\Gamma(y, Q^2, \kappa^2)$. In this section we calculate the higher order corrections to the new twist two structure functions $b_{1,2}^\Gamma(y, Q^2, \kappa^2)$ and the modified unpolarised structure functions $F_{1,2}^\Gamma(y, Q^2, \kappa^2)$. As the unpolarised structure functions $F_{1,2}^\Gamma(y, Q^2, \kappa^2)$ gets modified due to the virtuality, it is *a priori* not clear if the Hard Scattering Coefficients (HSC) are also modified. For completeness we also evaluate the higher order correction to $g_1^\Gamma(y, Q^2, \kappa^2)$.

Higher order corrections (both *em* and strong) are relevant to the study of $\gamma^* \Gamma \rightarrow \text{hadron}$ cross section since the photonic corrections go as $\ln Q^2$, whereas the QCD corrections turn out to be of leading order, since $f_{q/\Gamma} \sim \ln Q^2 \sim 1/\alpha_s(Q^2)$ (i.e. it compensates for additional power of $\alpha_s(Q^2)$). To go beyond the leading order we make use of the factorisation approach [10] which

is a field theoretical generalisation of the free field analysis. This approach can be employed for the photon targets also since its proof does not depend on the target but depends only on the underlying theory. This method ensures a systematic separation of hard and soft parts, *i.e.*

$$W_{\mu\nu}^{\gamma^*\Gamma(\epsilon)}(y, Q^2, \kappa^2) = \sum_{a,h} \int_y^1 \frac{dz}{z} f_{a(h)/\Gamma(\epsilon)}(z, \mu_R^2, \kappa^2) H_{a(h)\gamma^*}^{\mu\nu}(q, zp, \mu_R^2, \alpha_s(\mu_R^2), \alpha) + \dots, \quad (38)$$

where the hard part H of the processes are perturbatively calculable and the summation over a includes partons (quarks and gluons) and free photons. The ‘soft’ parts are defined below as photon matrix elements of bilocal quark, gluon and photon operators,

$$f_{\frac{q(\uparrow\downarrow)}{\Gamma(\epsilon)}}(z, \mu^2, \kappa^2) = \frac{1}{4\pi} \int d\xi^- e^{-iz\xi^-k^+} \langle \Gamma(k, \epsilon^*) | \bar{\psi}_a(0, \xi^-, 0_\perp) \gamma^+ \Lambda_\pm \mathcal{G}_b^a \psi^b(0) | \Gamma(k, \epsilon) \rangle_c, \quad (39)$$

$$f_{\frac{\bar{q}(\uparrow\downarrow)}{\Gamma(\epsilon)}}(z, \mu^2, \kappa^2) = \frac{1}{4\pi} \int d\xi^- e^{-iz\xi^-k^+} \langle \Gamma(k, \epsilon^*) | \bar{\psi}_a(0) \gamma^+ \Lambda_\mp \mathcal{G}_b^{a\dagger} \psi^b(0, \xi^-, 0_\perp) | \Gamma(k, \epsilon) \rangle_c, \quad (40)$$

$$f_{\frac{g}{\Gamma(\epsilon)}}(z, \mu^2, \kappa^2) = \frac{i}{4\pi z k^+} \int d\xi^- e^{-iz\xi^-k^+} \left[\langle k, \epsilon^* | F_a^{+\mu}(0, \xi^-, 0_\perp) \mathcal{G}_b^a F_\mu^{+b}(0) | k, \epsilon \rangle_c \right. \\ \left. + \langle \Gamma(k, \epsilon^*) | F_a^{+\mu}(0) \mathcal{G}_b^{a\dagger} F_\mu^{+b}(0, \xi^-, 0_\perp) | \Gamma(k, \epsilon) \rangle_c \right], \quad (41)$$

$$f_{\frac{\Delta g}{\Gamma(\epsilon)}}(z, \mu^2, \kappa^2) = \frac{i}{4\pi z k^+} \int d\xi^- e^{-iz\xi^-k^+} \left[\langle \Gamma(k, \epsilon^*) | F_a^{+\mu}(0, \xi^-, 0_\perp) \mathcal{G}_b^a \tilde{F}_\mu^{+b}(0) | \Gamma(k, \epsilon) \rangle_c \right. \\ \left. - \langle \Gamma(k, \epsilon^*) | F_a^{+\mu}(0) \mathcal{G}_b^{a\dagger} \tilde{F}_\mu^{+b}(0, \xi^-, 0_\perp) | \Gamma(k, \epsilon) \rangle_c \right], \quad (42)$$

where $\Lambda_\pm = (1 \pm \gamma_5)/2$ and $\mathcal{G}_b^a = \mathcal{P} \exp \left[ig \int_0^{\xi^-} d\zeta^- A^+(0, \zeta^-, 0_\perp) \right]_b^a$ (\mathcal{P} denotes the path ordering of gauge fields). For $\epsilon(0)$, the γ_5 term would not be present in the above definitions. The above definitions hold for the photon case also wherein the $SU(3)_c$ group indices will be absent. The gauge invariant definitions of quark, gluon and photon distributions defined above have a probabilistic interpretation of finding a parton or photon inside the target photon. These matrix elements are in principle calculable if we treat the photon as a point like perturbative object. But we know that the photon does not behave like a point like perturbative object at small energy scales. Hence we collect all the higher order effects as well as nonperturbative effects inside the matrix elements and treat them as theoretical inputs. The fact that they are calculable order by order in perturbation theory for point like targets such as quarks, gluons and photons is exploited in the evaluation of HSC.

The HSCs can be evaluated order by order using the factorisation formulae by replacing target photon by parton targets *i.e.*, quarks, gluons and real photons. We calculate the HSCs up to $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$. Let us first concentrate on the quark sector. The quark sector gets contribution to $\mathcal{O}(\alpha)$ by $\gamma^*(q) q(p) \rightarrow q(p')$ (Fig. 2a), $\mathcal{O}(\alpha^2)$ by $\gamma^*(q) q(p) \rightarrow q(k) \gamma(k')$ and

$\mathcal{O}(\alpha\alpha_s)$ by $\gamma^*(q) q(p) \rightarrow q(k) g(k')$ (Fig. 2b). For the photonic corrections, we replace gluon lines by photon lines in Fig. 2b. From the factorisation formulae it is clear that the calculation of HSCs involves the cross sections of the above mentioned processes as well as the matrix elements given in eqns. (39,40), with target photon replaced by quarks to appropriate orders. The contributions to various structure functions are extracted using the appropriate projection operators $P_{\mu\nu}^i$, where i runs over the various unpolarised, singly polarised and polarised structure functions. These projection operators are given in the Ref. [11, 12]. We define $W_{\gamma^*q}^i = P_{\mu\nu}^i W_{\gamma^*q}^{\mu\nu}$ (i.e., in eqn. (38) replace the target photon Γ with quark and evaluate the evaluate to $\mathcal{O}(\alpha\alpha_s)$). This corresponds to evaluation of the bremsstrahlung diagrams given in Fig. 2b. At large Q^2 , we get

$$\begin{aligned}
W_{\gamma^*q}^{F_1}(z, Q^2) &= 2f_c \alpha \left\{ -\left(\frac{1+z^2}{1-z}\right)_+ \ln \beta_g - 2 \frac{1+z^2}{1-z} \ln z + (1+z^2) \left(\frac{\ln(1-z)}{1-z}\right)_+ \right. \\
&\quad \left. + 2z + 1 - \frac{3}{2} \frac{1}{(1-z)_+} - \delta(1-z) \left(\frac{9}{4} + \frac{2\pi^2}{3}\right) \right\} , \\
W_{\gamma^*q}^{F_2}(z, Q^2) &= 2z \left\{ W_{\gamma^*q}^{F_1}(z, Q^2) + 2\alpha f_c 2z \right\} , \\
W_{\gamma^*q}^{b_1}(z, Q^2) &= 2 W_{\gamma^*q}^{F_1}(z, Q^2) , \\
W_{\gamma^*q}^{b_2}(z, Q^2) &= 2 W_{\gamma^*q}^{F_2}(z, Q^2) , \\
W_{\gamma^*q}^{g_1}(z, Q^2) &= W_{\gamma^*q}^{F_1}(z, Q^2) - 2\alpha f_c (1-z) ,
\end{aligned} \tag{43}$$

where $z = Q^2/2p \cdot q$, $\beta_g = m_g^2/Q^2$, $f_c = (4\alpha_s/3, \alpha)$ is the coupling factor depending on gluon or photon bremsstrahlung respectively and the subscript $+$ denotes the ‘+ function’ regularisation of the singularity as $z \rightarrow 1$. For the photons m_g will be replaced by m_γ . To avoid the mass singularity, we have kept the gauge bosons massive. If quark masses are also kept nonzero, then there would be both logarithmic and power singularities as these masses go to zero simultaneously. If one of the prescriptions is chosen, say massive gauge boson prescription, the singularities boil down to logarithmic singularities with an additional constant part which depends on the prescription. In addition there are some mass singularities coming from the virtual diagrams which are exactly canceled by those which arise from regulating the bremsstrahlung diagram in the limit $z \rightarrow 1$ (‘+ function’). We have considered the massive gauge boson prescription which has not so far been considered in the literature while calculating the corrections to structure functions. This is a convenient choice since we are dealing with the virtual photons. It has been customary to consider the massive quark prescription or the dimensional regularisation method to deal with the infrared (IR) singularities.

For the structure functions $F_{1,2}^\Gamma(y, Q^2, \kappa^2)$ and $b_{1,2}^\Gamma(y, Q^2, \kappa^2)$ the relevant matrix elements are obtained by replacing Γ in eqns. (39,40) by quarks, where only the vector operator will contribute. In the case of polarised structure function $g_1^\Gamma(y, Q^2, \kappa)$ the axial vector operator will contribute (eqns. (39, 40)). The contributing matrix elements are shown in Fig. 3. The Feynman rules for the eikonal lines and vertices are given in Ref. [10]. We regulate the ultraviolet (UV) divergences appearing in these diagrams using dimensional regularisation and keep gauge boson masses nonzero to regulate the mass singularities. Here too there is a similar cancellation of mass singularities among the virtual and real diagrams, leaving a logarithmic singularity and a prescription dependent constant as given below

$$\begin{aligned} f_q^{(1)}(z, \mu_R^2) &= \frac{f_c}{4\pi} \left\{ \left(\frac{1+z^2}{1-z} \right)_+ \ln \beta' + \frac{1+z^2}{1-z} \ln z + 2(1-z) - \delta(1-z) \left(\frac{9}{4} - \frac{\pi^2}{3} \right) \right\}, \\ f_{\frac{\Delta q}{q(h)}}^{(1)}(z, \mu_R^2) &= h f_q^{(1)}(z, \mu_R^2), \end{aligned} \quad (44)$$

where $\beta'_g = m_g^2/\mu_R^2$ and μ_R is the renormalisation scale. The superscript (1) in the above equations denotes that they are evaluated to order α or α_s as the case may be. This equivalence among the polarised and unpolarised matrix elements does not hold if we keep the quark masses also nonzero. Substituting the above matrix elements and cross sections (eqn. (43)) in the factorisation formulae for quark sector, we obtain

$$\begin{aligned} H_{q\gamma^*}^{F_1}(z, Q^2) &= 2\alpha f_c \left\{ \left(\frac{1+z^2}{1-z} \right)_+ \ln \frac{Q^2}{\mu_R^2} - \frac{1+z^2}{1-z} \ln z + (1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ \right. \\ &\quad \left. + 3 - \frac{3}{2(1-z)_+} - \delta(1-z) \left(\frac{9}{2} + \frac{\pi^2}{3} \right) \right\}, \\ H_{q\gamma^*}^{F_2}(z, Q^2) &= 2z \left[H_{q\gamma^*}^{F_1}(z, Q^2) + 2f_c \alpha 2z \right], \\ H_{q\gamma^*}^{b_1}(z, Q^2) &= 2H_{q\gamma^*}^{F_1}(z, Q^2), \\ H_{q\gamma^*}^{b_2}(z, Q^2) &= 2H_{q\gamma^*}^{F_2}(z, Q^2), \\ H_{q\gamma^*}^{g_1}(z, Q^2) &= H_{q\gamma^*}^{F_1}(z, Q^2) + 2\alpha f_c(z-1). \end{aligned} \quad (45)$$

Note that the mass term in the logarithmic and the prescription dependent constant term cancel among the cross section $W_{\gamma^*q}^i(z, Q^2)$ and the matrix element $f_{a/b}(z, Q^2)$, leaving behind the HSC independent of gauge boson mass.

Next we will discuss the corrections coming from the gluon initiated subprocesses. The contributing subprocesses are $\gamma^*(q) g(p) \rightarrow q(k) \bar{q}(k')$ (Fig. 4) and $\gamma^*(q) \gamma(p) \rightarrow q(k) \bar{q}(k')$. To calculate the gluonic and photonic HSCs we need in addition to the above cross sections, the

matrix element (eqns. (41,42)) between gluon states and between photon states respectively. The contributions to various structure functions from the cross section, at large Q^2 are

$$\begin{aligned}
W_{g\gamma^*}^{F_1}(z, Q^2) &= 8 \alpha f_c \left\{ (2z^2 - 2z + 1) \left(\ln \frac{\mathcal{M}_g^2}{Q^2} + \ln \frac{z}{1-z} \right) \right. \\
&\quad \left. - \frac{p^2}{\mathcal{M}_g^2} z(1-z) - \frac{2m^2}{\mathcal{M}_g^2} z(1-z) + 2z^2 - 2z + 1 \right\} , \\
W_{g\gamma^*}^{F_2}(z, Q^2) &= 2z \left\{ W_{g\gamma^*}^{F_1}(z, Q^2) + 8 \alpha f_c (4z^2 - 4z) \right\} , \\
W_{g\gamma^*}^{b_1}(z, Q^2) &= 2 \left(W_{g\gamma^*}^{F_1}(z, Q^2) - 16 \alpha f_c \frac{p^2}{\mathcal{M}_g^2} z^2(1-z)^2 \right) , \\
W_{g\gamma^*}^{b_2}(z, Q^2) &= 2 \left(W_{g\gamma^*}^{F_2}(z, Q^2) - 32z \alpha f_c \frac{p^2}{\mathcal{M}_g^2} z^2(1-z)^2 \right) , \\
W_{g\gamma^*}^{g_1}(z, Q^2) &= 4 \alpha f_c \left\{ (1-2z) \left(\ln \frac{\mathcal{M}_g^2}{Q^2} + \ln \frac{z}{1-z} - \frac{p^2}{\mathcal{M}_g^2} z(1-z) + 1 \right) + \frac{2m^2}{\mathcal{M}_g^2} (1-z) \right\} ,
\end{aligned} \tag{46}$$

where $\mathcal{M}_g^2 \equiv m^2 - p^2 z(1-z)$. Observe that the new singly polarised structure functions $b_{1,2}^\Gamma(z, Q^2, \kappa^2)$ differs from $F_{1,2}^\Gamma(z, Q^2, \kappa^2)$ by a term $(2p^2 z^2(1-z)^2/\mathcal{M}_g^2)$. This extra term is due to the gauge boson of scalar polarisation, as a result of the off-shell nature of the gauge boson. In the parton model, care should be taken in using the above result for the subprocess cross sections because the unphysical scalar polarisation gluon should not be considered. In this procedure to calculate the HSCs, this step is just a technique and we have no reason to avoid the scalar polarisation. As we go along, it will become clear that this unphysical degree of freedom will have no effect on the HSC. The relevant matrix elements are

$$\begin{aligned}
f_g^{(1)}(z, \mu_R^2) &= \frac{f_c}{4\pi} \left[(2z^2 - 2z + 1) \ln \frac{\mathcal{M}_g^2}{\mu_R^2} - \frac{2m^2}{\mathcal{M}_g^2} z(1-z) - \frac{p^2}{\mathcal{M}_g^2} z(1-z) + 2z(1-z) \right] , \\
f_{\delta g}^{(1)}(z, \mu_R^2) &= f_g^{(1)}(z, \mu_R^2) - \frac{f_c}{4\pi} \left[2 \frac{p^2}{\mathcal{M}_g^2} z^2(1-z)^2 \right] , \\
f_{\frac{\Delta q}{g(h)}}^{(1)}(z, \mu_R^2) &= h \frac{f_c}{4\pi} \left[(1-2z) \ln \frac{\mathcal{M}_g^2}{\mu_R^2} + \frac{p^2}{\mathcal{M}_g^2} z(1-z) \right] ,
\end{aligned} \tag{47}$$

which are evaluated from the cut diagrams shown in Fig. 5. The details of the calculation can be found in Ref. [11] for polarised case and in Ref. [12] for the unpolarised case. Substituting the cross section and matrix elements in the factorisation formulae, we get

$$\begin{aligned}
H_{g\gamma^*}^{F_1}(z, Q^2) &= 8\alpha f_c \left[(2z^2 - 2z + 1) \left(-\ln \frac{Q^2}{\mu_R^2} + \ln \frac{z}{1-z} \right) + 4z^2 - 4z + 1 \right] , \\
H_{g\gamma^*}^{F_2}(z, Q^2) &= 2z \left[H_{g\gamma^*}^{F_1}(z, Q^2) + 8\alpha f_c (4z^2 - 4z) \right] ,
\end{aligned}$$

$$\begin{aligned}
H_{g\gamma^*}^{b_1}(z, Q^2) &= 2H_{g\gamma^*}^{F_1}(z, Q^2) , \\
H_{g\gamma^*}^{b_2}(z, Q^2) &= 2H_{g\gamma^*}^{F_2}(z, Q^2) , \\
H_{g\gamma^*}^{g_1}(z, Q^2) &= 4\alpha f_c \left[(2z-1) \left(\ln \frac{Q^2}{\mu_R^2} - \ln \frac{z}{1-z} \right) - 4z + 3 \right] .
\end{aligned} \tag{48}$$

As expected the mass terms cancel among cross section and matrix element, so also the scalar gauge boson contribution. Substituting the calculated HSCs in eqn. (38) one gets the QCD corrected $W_{\mu\nu}^\Gamma(y, Q^2, \kappa^2)$ for finite virtual target photon mass. Choose the renormalisation scale $\mu_R^2 = Q^2$ so that the Q^2 dependency of the HSCs will be transferred to the strong coupling constant and the parton distribution function $f_{a/\Gamma}(z, \mu_R^2 = Q^2, \kappa^2)$. At every order in $\alpha_s(Q^2)$, the $\ln Q^2$ growth of $f_{a/\Gamma}(Q^2)$ is compensated for by the coupling constant. The photonic operator also contributes in the same order. But due to the $\ln Q^2$ growth of $f_{q/\Gamma}(Q^2)$, the photon structure tensor $W_{\mu\nu}^\Gamma$ grows as $\ln Q^2$ to leading order (α_s^0). The first moments of the gluonic and photonic HSC of $g_1^\Gamma(z, Q^2, \kappa^2)$ vanish and hence are not corrected by these operators. So the first moment of $g_1^\Gamma(z, Q^2, \kappa^2)$ is proportional to only quark field operators i.e. $f_{\Delta q/\Gamma}(Q^2, \kappa^2)$. For off-shell target photons this quantity is nonzero.

5 Conclusion

We have analysed the $\gamma^*\Gamma \rightarrow X$ subprocess of the $e^+e^- \rightarrow e^+e^-X$ process in the DIS limit. Our analysis correctly includes the virtuality of the target photon by taking into account the contribution coming from the scalar polarisation. The virtuality of the target photon gives rise to four new singly polarised structure functions. Two of these new structure functions are found to be of twist two. Hence we find that the unpolarised cross section is modified by these twist two structure functions to leading order. In addition the usual $e \rightarrow \gamma$ splitting functions are altered by the scalar polarisation of the virtual target photon.

Using the free field analysis we have studied the physical interpretation of the new structure functions $b_{1,2}^\Gamma(y, Q^2, \kappa^2)$ and we also find that the physical interpretation of unpolarised structure functions $F_{1,2}^\Gamma(y, Q^2, \kappa^2)$ are modified by the scalar polarisation. In the process we have obtained relations among various structure functions. The free field analysis is useful to show that our results are consistent with the existing real photon results in the limit $\kappa^2 \rightarrow 0$.

In addition we have also systematically computed various QCD and QED contributions to these structure functions using the factorisation method. We have redone this because the old results are modified by the existence of extra polarisation state *viz.* parton distribution function

in a scalar polarised photon. The differential cross section is found to grow as $\ln Q^2$ while higher order QCD corrections are found to contribute to leading order. Interestingly, the first moment of $g_1^e(y, Q^2)$ is proportional only to the first moment of the quark operator $f_{\Delta q/\Gamma}(Q^2, \kappa^2)$.

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Appendix

The unpolarised cross section in eqn. (12) can be derived from eqn. (4) by summing over the polarisation states of the positron, as

$$\frac{d\sigma^{s_1(\uparrow+\downarrow)}}{dx dQ^2} = \frac{\alpha^2}{4x^2 s^2 Q^2} L^{\mu\mu'}(q, p_1, s_1) \left(W_{\mu\mu'}^e(q, p_2, \uparrow) + W_{\mu\mu'}^e(q, p_2, \downarrow) \right). \quad (49)$$

From eqn. (5), we have

$$\begin{aligned} W_{\mu\mu'}^e(x, Q^2, \uparrow) + W_{\mu\mu'}^e(x, Q^2, \downarrow) &= \alpha x \int \frac{d\kappa^2}{\kappa^2} \int_x^1 \frac{dy}{y^2} \left(L^{\nu\nu'}(k, p_2, \uparrow) + L^{\nu\nu'}(k, p_2, \downarrow) \right) \\ &\times \sum_{\lambda=0, \pm 1} g^{\lambda\lambda} \epsilon_\nu^*(k, \lambda) \epsilon_{\nu'}(k, \lambda) W_{\mu\mu'}^\Gamma(q, k, \lambda). \end{aligned} \quad (50)$$

Since the combination $(L^{\nu\nu'}(k, p_2, \uparrow) + L^{\nu\nu'}(k, p_2, \downarrow))$ is symmetric in $\nu\nu'$ only the symmetric part of $\epsilon_\nu^*(k, \lambda)\epsilon_{\nu'}(k, \lambda)$ gets projected. Hence

$$\begin{aligned} \sum_{\lambda=0, \pm 1} g^{\lambda\lambda} \epsilon_\nu^*(k, \lambda) \epsilon_{\nu'}'(k, \lambda) W_{\mu\mu'}^\Gamma(\lambda) &= \epsilon_\nu^*(k, 0) \epsilon_{\nu'}(k, 0) W_{\mu\mu'}^\Gamma(0) \\ &- \epsilon_\nu^*(k, 1) \epsilon_{\nu'}(k, 1) \left(W_{\mu\mu'}^\Gamma(1) + W_{\mu\mu'}^\Gamma(-1) \right), \end{aligned} \quad (51)$$

where we have used the fact that the symmetric part of $\epsilon^*(k, 1)\epsilon(k, 1) = \epsilon^*(k, -1)\epsilon(k, -1)$. In the above equation $W_{\mu\mu'}^\Gamma(0)$ and $W_{\mu\mu'}^\Gamma(1) + W_{\mu\mu'}^\Gamma(-1)$ are both symmetric and hence consists of two parts *viz.* unpolarised and singly polarised (eqn. (9)). These are denoted by the superscripts *UP* and *SP*.

Now we make use of the following properties of the unpolarised and singly polarised sector (eqn. (9)),— (i) the unpolarised part is independent of the polarisation of the photon

$$W_{\mu\mu'}^{\Gamma(UP)}(0) = W_{\mu\mu'}^{\Gamma(UP)}(1) = W_{\mu\mu'}^{\Gamma(UP)}(-1), \quad (52)$$

and (ii) sum of polarisation of the singly polarised part is

$$\sum_{\lambda=0, \pm 1} g^{\lambda\lambda} W_{\mu\mu'}^{\Gamma(SP)}(\lambda) = 0. \quad (53)$$

The above two properties implies

$$\sum_{\lambda=0, \pm 1} g^{\lambda\lambda} W_{\mu\mu'}^\Gamma(\lambda) = -W_{\mu\mu'}^{\Gamma(UP)}, \quad (54)$$

and

$$\sum_{\lambda=0, \pm 1} C(\lambda) W_{\mu\mu'}^\Gamma(\lambda) = W_{\mu\mu'}^{\Gamma(SP)}(0). \quad (55)$$

Making use of these properties we get

$$\begin{aligned}
\sum_{\lambda=0\pm1} g^{\lambda\lambda} \epsilon_{\nu}^*(k, \lambda) \epsilon'_{\nu}(k, \lambda) W_{\mu\mu'}^{\Gamma}(\lambda) &= - \sum_{\lambda=0,\pm1} g^{\lambda\lambda} \epsilon_{\nu}^*(k, \lambda) \epsilon'_{\nu}(k, \lambda) \sum_{\lambda=0,\pm1} g^{\lambda\lambda} W_{\mu\mu'}^{\Gamma}(\lambda) \\
&+ \frac{1}{2} \sum_{\lambda=0\pm1} C(\lambda) \epsilon_{\nu}^*(k, \lambda) \epsilon'_{\nu}(k, \lambda) \sum_{\lambda=0,\pm1} C(\lambda) W_{\mu\mu'}^{\Gamma}(\lambda). \quad (56)
\end{aligned}$$

Substituting the above equation in eqn. (50) we reproduce the unpolarised cross section eqn. (12).

As the scalar polarisation of the virtual photon does not affect the polarised cross section, the derivation of the polarised cross section is the same as in the real photon case and hence is not repeated.

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Table. 1

Table. 1 Various structure functions related to scaling functions and their n^{th} moments.

Structure Function	Scaling Functions	Moments
$F_1(y)$	$\tilde{A}(y)$	$\frac{(-i)^{n-1}}{4} A_{n-1}$
$F_2(y)$	$2y\tilde{A}(y)$	$\frac{(-i)^n}{2} A_n$
$b_1(y)$	$-\frac{i}{2}y\frac{\partial^2}{\partial y^2}\tilde{B}(y)$	$\frac{(-i)^{n-1}}{4} B_{n-1}$
$b_2(y)$	$-iy^2\frac{\partial^2}{\partial y^2}\tilde{B}(y)$	$\frac{(-i)^n}{2} B_n$
$b_3(y)$	$-iy\frac{\partial}{\partial y}\tilde{B}(y)$	$-\frac{(-i)^n}{2(n+1)} B_n$
$b_4(y)$	$-i\tilde{B}(y)$	$\frac{(-i)^n}{2n(n+1)} B_n$
$g_1(y)$	$-\frac{1}{2}y\frac{\partial}{\partial y}\tilde{C}(y)$	$\frac{(-i)^{n-1}}{4} C_{n-1}$
$g_2(y)$	$\frac{1}{2}\tilde{C}(y) + \frac{1}{2}y\frac{\partial}{\partial y}\tilde{C}(y)$	$-\frac{(-i)^{n-1}}{4} \frac{n-1}{n} C_{n-1}$

Figure Captions

Fig 1. The process $e^+e^- \rightarrow e^+e^-X$ via the photon-photon interaction.

Fig 2. (a) Born diagram, (b) Next to leading order corrections to it.

Fig 3. The contribution to the matrix element $f_{q/q}$ and $f_{\Delta q/q}$ up to $\mathcal{O}(\alpha_s)$.

Fig 4. The gluon (photon) produced in the target photon interacting with the probe photon at a higher order.

Fig 5. The $\mathcal{O}(\alpha_s)$ contribution to the matrix element $f_{q/g}$ and $f_{\Delta q/g}$. The vertex is γ^+ for unpolarised and $\gamma^+\gamma_5$ for polarised. Here only the nonvanishing diagrams are given.